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Thermal convection and surface temperatures in porous media

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Abstract—Experimental steady-state temperature measurements in a porous medium containing a buried heater are fitted to a conventional thermal convection model. A constant heat transfer coefficient at the free surface is shown to adequately describe the boundary condition. A simple one-dimensional model uniquely relating power input to surface temperature irrespective of permeability, source size and depth is derived. This is shown, to the degree required to monitor self-heating in stockpiles, to be consistent with the experiments and the general model.

INTRODUCTION

MUCH WORK on thermally driven convection in porous media has been published. A recent article by Cheng [1] reviews much of the literature. The early work was by Lapwood [2], who analysed the stability of free convection of fluids in horizontal porous layers heated from below. Lapwood made use of the Darcy law and the Boussinesq approximation and assumed the temperatures of both the upper and lower surfaces to be known. An approximate analysis was applied to the simplified equations to show that there existed a critical value of the Rayleigh number, below which no convection occurred.

More recent work by Bejan [3] dealt with flow and temperature distributions about a concentrated energy source in an infinite porous medium. An approximate analysis of the model equations made comparisons between coupled convection/conduction and pure conduction temperature profiles. This clearly demonstrated the effect of convection, which is to distort temperature profiles from those of pure conduction, shifting them locally in the direction of the flow.

Bejan [4] also investigated the phenomenon of lateral penetration of convection into the vertical sides of heated porous media. The two-dimensional model was shown to be essentially one-dimensional due to the large difference in the natural horizontal and vertical scales. Approximate analysis of the one-dimensional model gave information on the amounts of penetration to be expected under various circumstances; this was illustrated by application of the results to a helium-cooled winding.

Further work by Bejan [5] on vertical penetration into the base of porous structures caused by local surface temperature variations (hot or cold spots) examined the degree of penetration of flow and temperature disturbances.

Young *et al.* [6] examined the role of natural convection in spontaneous combustion of coal stockpiles. Measurements of temperatures in packed beds of coal containing a buried line heater were made. This bed was modelled using the Darcy law-modified Navier–Stokes equations and the Boussinesq approximation. In this work, Young assumed the temperature of the top surface of the coal bed to be approximately ambient, which for a line heater gave acceptable results.

In all of the work mentioned thus far, the porous bodies have been infinite (no surfaces) [3], or the surface temperatures have been assumed to be known, or of known spatial variation [2, 4–6], and in every case the physical geometry is well defined. However, in practice, surface temperatures are seldom explicitly known, but rather take on values dependent on internal temperatures or power dissipation. This necessitates the use of boundary conditions dependent on the local surface temperature and temperature gradients, known as Cauchy boundary conditions. The use of a Cauchy boundary condition on exposed surfaces of the bed is examined in this work.

One instance where surface temperatures are not constant or specifiable occurs when combustion is taking place within piles of coal. In fact, regions on the surface of the pile with temperatures higher than ambient (so-called surface 'hot spots') may indicate the presence of combustion within the bed. The effects of various parameters such as coal reactivity, particle size, wind, etc. on the liability of a dump to combustion have been discussed by Brooks and Glasser [7]. It would clearly be unacceptable to assume ambient surface temperatures in modelling situations of this kind. Since surface temperature is an indication of the condition of the interior of a dump, a relation

NOMENCLATURE

A area [m^2]
Bi Biot number
d_p nominal particle diameter [m]
g acceleration due to gravity [m s^{-2}]
G dimensionless power input, $QR^2/(k_b T_0 V_s)$
h film coefficient of heat transfer [W m^{-2} K^{-1}]
H_q height of source from reference level [m]
k_b effective thermal conductivity of medium [W m^{-1} K^{-1}]
K permeability [m^2]
L thickness of layer [m]
P pressure [Pa]
Q power [W]
q power per unit area [W m^{-2}]
r radial coordinate [m]
R maximum radial dimension of bed [m]
Ra Rayleigh number, $KgL\rho_0/(\mu\alpha)$
Ra_1 Rayleigh number, $\beta T_0 Ra$
T temperature [K]
V_s heater volume [m^3]
v velocity [m s^{-1}]
v^* dimensionless velocity
Z vertical coordinate [m].

Greek symbols
α thermal diffusivity, $k_b/(\rho_g^0 C_{p,g})$ [m^2 s^{-1}]
β coefficient of thermal expansion of air [K^{-1}]
ζ dimensionless vertical coordinate, z/Z
Θ dimensionless temperature $(T-T_0)/T_0$
μ viscosity [kg m^{-1} s^{-1}]
ξ dimensionless radial coordinate, r/R
ρ density [kg m^{-3}]
Ψ dimensionless stream function.

Superscripts and subscripts
0 ambient conditions
g gas
s surface conditions
z vertical direction.

between surface temperature and internal power input would be most useful for the identification and grading of combustion hazards as early as possible, facilitating prompt action.

EXPERIMENTAL

Figure 1 shows the experimental test section and power source arrangement used in ref. [8]. The positions of the thermistors in the bed are shown in Fig. 2. The heater was an upright cylinder and had a diameter of 0.035 m and a height of 0.06 m. The porous medium used was coal which had been screened to a nominal size. For each coal size a range of power inputs was used and then the coal was replaced by another nominal size. The coal was hand-packed and the bed kept under an atmosphere of nitrogen to prevent combustion.

Experimental details of the bed with the line-heater are discussed in ref. [6]. In both cases [6, 8], 45 thermistors were used. Readings were taken every half-hour, and when no discernible change in temperature could be noted, the system was deemed to be at steady state; time taken to reach steady state was 5 days on average.

MODEL FORMULATION

The dimensionless stream function and energy equations describing free convection within a porous body with cylindrical geometry and radial symmetry

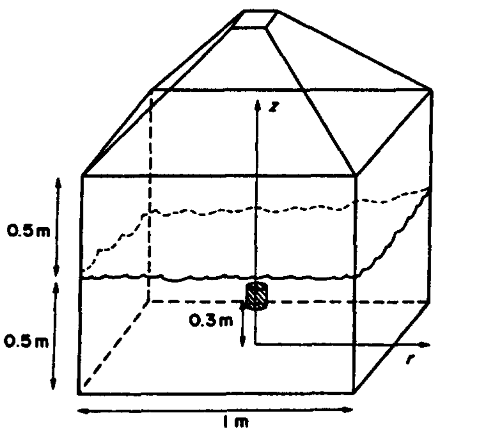

FIG. 1. Diagram of experimental apparatus. The cylindrical power source is shaded, and the lower half of the bed is packed with coal.

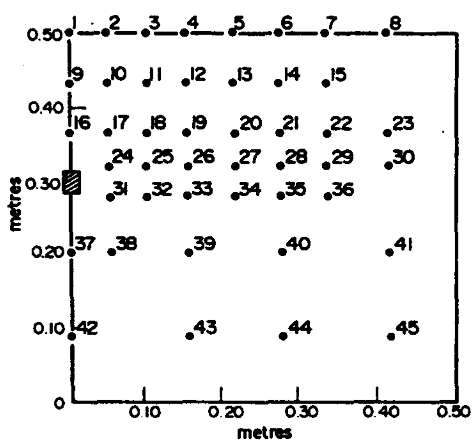

FIG. 2. Diagram of a vertical cross-section through the bed, showing positions of the thermistors.

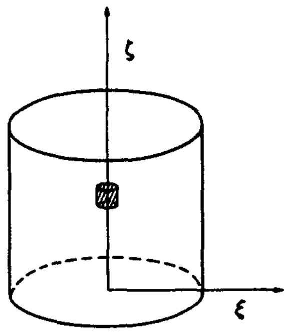

FIG. 3. Cylindrical geometry in dimensionless coordinates. Power source on centreline is shaded.

(see Fig. 3) have been derived without the energy source term in ref. [9]. Allowing for the energy source they are as follows:

$$\frac{\partial^2\Psi}{\partial\xi^2}-\frac{1}{\xi}\frac{\partial\Psi}{\partial\xi}+\frac{\partial^2\Psi}{\partial\zeta^2}=\xi\,Ra_1\frac{\partial\Theta}{\partial\xi} \quad (1)$$

$$\left[\frac{1}{\xi}\frac{\partial\Psi}{\partial\zeta}\right]\frac{\partial\Theta}{\partial\xi}-\left[\frac{1}{\xi}\frac{\partial\Psi}{\partial\xi}\right]\frac{\partial\Theta}{\partial\zeta}-\nabla^2\Theta=G. \quad (2)$$

The attendant dimensionless boundary conditions are as follows:

$$\Psi(0,\zeta)=\Psi(1,\zeta)=0 \quad 0\leqslant\zeta\leqslant 1 \quad (3)$$

$$\Psi(\xi,0)=0 \quad 0\leqslant\xi\leqslant 1 \quad (4)$$

$$\left.\frac{\partial\Psi}{\partial\zeta}\right|_{\zeta=1}=0 \quad 0\leqslant\xi\leqslant 1 \quad (5)$$

$$\left.\frac{\partial\Theta}{\partial\xi}\right|_{\xi=0}=0 \quad 0\leqslant\zeta\leqslant 1 \quad (6)$$

$$\left.\frac{\partial\Theta}{\partial\zeta}\right|_{\zeta=1}=-Bi\,\Theta \quad 0\leqslant\xi\leqslant 1 \quad (7)$$

$$\Theta(\xi,0)=0 \quad 0\leqslant\xi\leqslant 1 \quad (8)$$

$$\Theta(1,\zeta)=0 \quad 0\leqslant\zeta\leqslant 1. \quad (9)$$

The Boussinesq approximation [10], Darcy law [11] and stream function [12], as well as various dimensionless quantities (see Nomenclature) have been used to arrive at this result. An analogous set of equations for Cartesian coordinates is described in ref. [6].

Equation (1) is the stream function form of the Darcy law-modified Navier–Stokes equation, and (2) is the convective–conductive energy transfer equation with an energy source term G. Boundary conditions (3) and (4) represent the stream function equivalent of the zero velocity condition on solid walls. Equation (5) specifies that the direction of flow in or out of the free top surface is normal to that surface. Equation (6) is the centreline symmetry condition on the temperature. Experimental observation [8] has confirmed that the temperatures on the base ($\zeta = 0$) and on the outer vertical surface ($\xi = 1$) are effectively ambient and equations (8) and (9), respectively, reflect these boundary conditions.

Equation (7) (where Bi may be a function of temperature) is the non-dimensional form of the Cauchy condition, namely

$$-k_b\frac{\partial T}{\partial z}=h(T-T_a).$$

This boundary condition equates energy transfer by conduction in the bed at the surface with energy transfer by conduction across a film of air just above the surface. Note that no boundary condition on energy transfer by convection is required. Convection transfer at the top surface is automatic due to the convective terms in the energy equation and the equation of material conservation for air, which is embedded in the stream function formulation above.

The heat transfer coefficient h need not be a constant and might, for instance, be a function of the surface temperature, or local convection velocity and direction. The following forms of h were proposed and tested. Each of these was applied to the whole surface, and later only to sections where flow left the bed. Type 1: h is a constant, fitted parameter. Type 2: h is a function of local temperature. Following results for convection from flat, horizontal plates [13], the following functional form for h is proposed:

$$h=h_0\left(\frac{\Delta T_s}{T_0}\right)^{0.25}$$

where h_0 is a fitted parameter. Type 3: h is allowed to depend on the local exit velocity, with the reasoning that the exiting air could create an insulating blanket, or drag cooler air into its wake, thereby increasing or decreasing the effective heat transfer. The following form is proposed:

$$h=h_0(1+h_1v_z)$$

where h_0 (positive) and h_1 (positive or negative) are fitted parameters.

SOLUTION OF THE EQUATIONS AND PARAMETER ESTIMATION

The non-linear two-dimensional model equations were solved simultaneously using FEPDE [14], a generalized implementation of an iterative finite-element Galerkin method [15]. The method requires an initial guess, which may be supplied by the user, and iterates until successively calculated values are within a user specified tolerance. The equations were solved on successively finer element meshes, until the solutions at two successive meshes were acceptably similar. Both four-noded (bilinear shape functions) and eight-noded (quadratic shape functions) quadrilateral elements were used. A single iteration (including assembly of matrices and solution of the resulting equations) on a 14 × 14 bilinear element mesh (450 degrees of freedom) required approximately 5 CPU seconds on an IBM 3073 model J24 mainframe computer. A complete calculation to convergence, for

$Ra_t = 400$ on the above mentioned mesh, required approximately 60 CPU seconds, the exact amount varying with the tolerance. The convergence criterion is shown below:

$$\left| \frac{u_{i+1} - u_i}{u_i} \right| \leqslant 0.001$$

where u is the variable (Θ or Ψ), and i is the iteration number. When this criterion was satisfied at all nodes for both variables, the calculation was deemed to have converged.

The effective thermal conductivity of a coal bed has been estimated previously [6] to be approximately 0.16 $W\ m^{-1}\ K^{-1}$ for the coal particle size and temperature range of interest. Thus only Ra_t and Bi needed to be estimated. This was done by minimizing the sum of squared differences between measured and predicted temperatures. A quasi-Newton method (ZXMIN) was used to vary the parameters and locate the minimum; an initial estimate of permeability was calculated (using the laminar term in the Ergun equation) to be $K \approx O\ (10^{-7}\ m^2)$ for $d_p \approx O\ (0.01\ m)$. The initial estimate of the film-coefficient parameters in all cases was zero.

SOLUTION CONDITIONS

The arrangement shown in Fig. 1 was modelled as an upright cylinder of radius and height 0.5 m, with a power source on the centreline, 0.3 m from the base, as idealized in Fig. 3. This approximation is justified by the fact that power inputs were low enough so that the maximum temperature at a radial distance of 0.4 m from the source never differed from ambient temperatures by more than 0.5°C [8]. To test the validity of the approximation, the model was also solved over a region of radius 0.71 m, corresponding to the centre-to-corner horizontal distance of the apparatus. The results were not noticeably different from those for a radius of 0.5 m, so the approximation appears to be acceptable. This is not surprising, since flow near the edges is very low and hence has a negligible effect, and the thermal conductivity of the bed is very small.

Model solutions obtained assuming the heater to be porous were not appreciably different from those obtained for a solid heater. The model equations were also solved allowing for variation of physical properties of the gas; however, the solution was again not appreciably different from that obtained using constant properties. Thus all further work was done assuming the heater to be porous, and the gas to have constant physical properties.

It was also shown that the Type 2 and 3 models for the heat transfer coefficients showed no improvement over that for Type 1 and so only the results for the constant value of h model (Type 1) are given.

RESULTS

Plots of the temperature contours and streamlines calculated using the best-fit parameters (for the case of a constant, fitted h, and a 10 W power source in a bed of 20 mm particles) are shown in Figs. 4(a) and (b), respectively. Also shown in Fig. 4(a) are experimental temperature contours, enabling comparison with model temperatures.

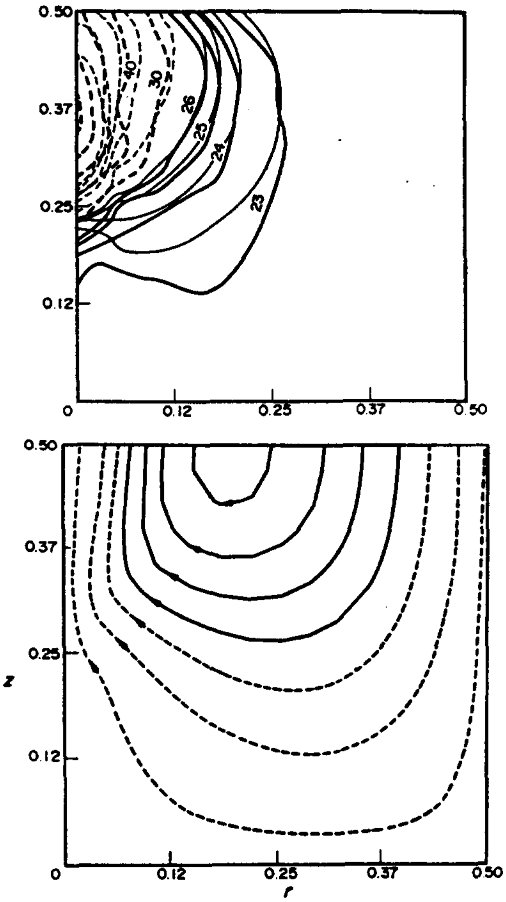

FIG. 4. (a) Model predicted isotherms for $Q = 10$ W, $d_p = 20$ mm. Contour intervals are 1°C for solid lines (starting at 23°C) and 10°C for broken lines (starting at 30°C). Bold lines are experimental contours, faint lines are model contours. (b) Model predicted streamlines for $Q = 10$ W, $d_p = 20$ mm.

An alternative comparison of model and experimental temperatures for the conditions of Fig. 4 is shown in Fig. 5. A perfect fit would yield a line through the origin, with a slope of 1; this line is superimposed on Fig. 5. As expected, the points are distributed about the line. Since temperatures near the source on the centreline would have the largest errors due to invalidity of the Boussinesq assumption and the assumption of the porosity of the heater, the temperatures measured at thermistors 9, 16 and 37 were given a weighting of zero when calculating the sum of squared temperature errors. This prevented a few, high temperatures from distorting the parameters unduly, but the result is that at the higher temperatures the model temperatures are consistently a little high.

Figure 6 shows a plot of the measured and predicted

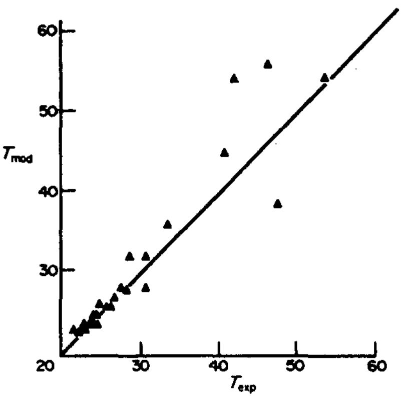

FIG. 5. Plot of model predicted temperatures vs measured temperatures for $Q = 10$ W, $d_p = 20$ mm.

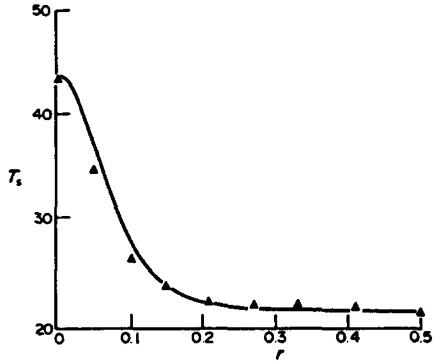

FIG. 6. Plot of surface temperatures for $Q = 7.5$ W, $d_p = 20$ mm. Point symbols represent measurements, solid line shows model prediction.

Table 1. Fitted parameters from experimental results

Q (W)	K ($m^2 \times 10^7$)	h ($W\,m^{-2}\,K^{-1}$)	Sum of squared errors (K^2)	
Cylindrical heater				
5.0	≈ 0†	1.7	222	$d_p < 0.005$ m
7.5	≈ 0†	0.8	491	
10.0	≈ 0†	1.6	649	
5.0	1.67	10.8	169	$d_p = 0.01$ m
10.0	1.43	9.6	427	
18.0	1.31	8.3	1188	
5.0	2.93	3.5	94	$d_p = 0.02$ m
7.5	2.67	4.0	232	
10.0	2.30	3.1	471	
Line heater				
15.0	1.50	6.8	307	$d_p = 0.01$ m
20.0	1.40	6.9	538	
30.0	1.90	8.9	853	

† Fitted permeabilities were so small that results were indistinguishable from pure conduction results (i.e. K identically zero).

surface temperatures for a power source of 7.5 W in a bed of 20 mm particles. From these results it is clear that the introduction of a heat transfer coefficient represents a substantial improvement over the assumption of a constant surface temperature.

Table 1 summarizes the results of the parameter

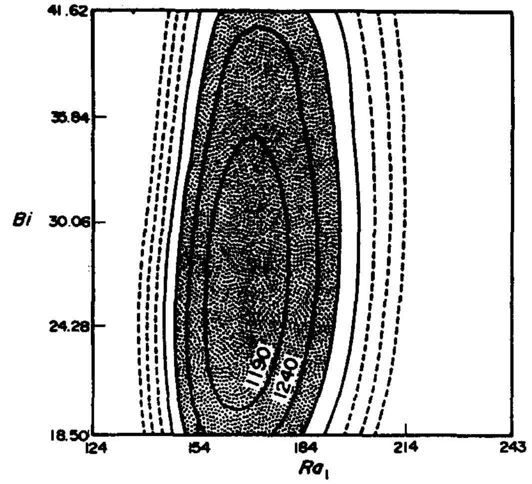

FIG. 7. Contours of the sum of squared temperature errors surface, $Q = 18$ W, $d_p = 10$ mm. Contour intervals are 50 $(°C)^2$. The approximate 90% confidence interval is shaded.

estimation in the form of permeability, K, and heat transfer coefficient, h, of Type 1. Note that apart from the entries marked with a dagger, the estimated permeabilities are all $\approx O\ (10^{-7}\ m^2)$ as predicted by the preliminary estimate. However, permeability is not directly related to nominal particle size, but rather to the distribution of particle sizes, thus it is understandable that the permeability figures for $d_p = 0.02$ m in Table 1 are not four times greater than those for $d_p = 0.01$ m, as is predicted by the Ergun correlation [16] using the nominal particle sizes.

Since the bed was not disturbed between the runs for the different power inputs, the permeabilities fitted to runs using the same particle size should be identical; in fact for a given particle size, parameters could be fitted to all three sets of results simultaneously. This would ensure that a single value of permeability is associated with a particular bed. However, the variation of fitted permeability within each particle-size class is acceptably small, and the average value for a given size range is accepted.

In order to judge the quality of the fits, the sum of squared errors surface for each experiment was calculated over a range of parameters spanning the optimum. The contours of each of these surfaces were drawn, and the 90% confidence intervals [17] estimated. Figure 7 shows the contours and the 90% confidence interval (shaded) for the parameters fitted to the 18 W power, 10 mm particle experiment.

It can be seen that the variation of Ra_l (and hence permeability) over the 90% confidence interval is not excessive. This is not the case with Bi (equivalently the film coefficient), where the upper value is more than 100% higher than the lower value. Thus not much confidence can be placed in the values of h in this case. The fitting of parameters was also repeated for h of Types 2 and 3. Neither the parameters nor the confidence intervals differed significantly from those given above, so the results are not presented.

As the contours in Fig. 7 are approximately ellipses with their major and minor axes aligned with those of the parameters, there is little covariance between the estimate of Ra_1 and h and the estimate of the Rayleigh number is reasonably accurate. It may be concluded that convection heat transfer is modelled with acceptable accuracy.

RELATING SURFACE TEMPERATURE MEASUREMENTS TO INTERNAL ENERGY DISSIPATION

A model capable of predicting internal power input from surface temperatures on real stockpiles is of use in monitoring these stockpiles for self-heating. In order to derive such a model it is necessary that the surface temperature be uniquely related to internal energy generation.

Furthermore, the model must be applicable despite the fact that the permeability of the bed is not generally known. Since stockpiles are generally inhomogeneous, permeability will vary with position, and the number and depth of the combusting regions, if any, are not known.

Because of these factors, detailed numerical modelling of the actual bed is not possible in this case. In order to develop a suitable model it should be recognized that whatever the source, all energy transfer must essentially take place through the top surface and this transfer is due to both convection and conduction across a film on the surface. It might be expected that to a first approximation, this transfer will be close to being directly proportional to the difference between surface and ambient temperature for both of these mechanisms. The depth and nature of the source is not likely to significantly influence this result. Furthermore, since the thermal conductivity of packed beds is generally low, it is also expected that transfer between streamlines might be small. Thus, a separate flow model for each of the streamlines is indicated. The hypothesis is that by integrating the measured temperature over the surface of the stockpile one can obtain an estimate of the power input in the bed.

A one-dimensional model is developed in the Appendix which enables an investigation of the variation of surface temperature with power input, permeability, film coefficient and source depth. The model which is depicted in Fig. 8 is one-dimensional and assumes that air at ambient temperatures enters a porous layer at the bottom and that there is also a flux of energy but none is lost out of the bottom. The flow is driven by the thermal convection induced by the energy input.

The solution, which essentially relates temperature to power input, is

$$v^* = Ra\left[1-\int_0^1 \frac{d\xi}{1+\Theta(\xi, v^*, q, Bi)}\right] \quad (10)$$

where

$$\Theta(\xi, v^*, q, Bi) = \frac{q}{v^*}\left(1-\frac{Bi}{v^*+Bi}e^{v^*(\xi-1)}\right). \quad (11)$$

The variation of surface temperature difference, ΔT_s, with power input per unit area, q, for various values of Ra and Bi was investigated as described below.

For a range of values of Ra, Bi and q, equation (10) (with equation (11)) was solved for v^* (considered constant, due to the Boussinesq approximation). Due to the transcendental nature of the equation, this was done numerically. Thus, Θ could be evaluated at any point in the layer (substitution of v^* into equation (11)). Since inner temperatures are of little interest in this case, only the temperature at the upper surface ($\xi = 1$) was evaluated.

These results from the model are summarized as a plot of q vs ΔT_s for various Ra and various Bi in Fig. 9 (for $h = 3$, 7 and 11 W m^{-2} K^{-1}, each for $Ra = 40$ and 4000). For each of the three pairs, the solid curve is for $Ra = 40$, the dotted curve for $Ra = 4000$.

There is little difference between the curves for different Ra, even over the large range shown. Thus, for a given h, the surface temperature of a one-dimensional porous layer can be taken to depend only on the power input over the range of Ra of practical interest. As a result of this, neither the layer thickness nor permeability are required, since both of these are embedded in Ra. Thus, for a given situation,

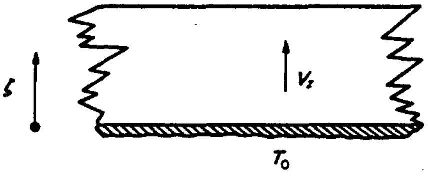

FIG. 8. Cross-section of a one-dimensional horizontal porous layer. The porous heater is shaded.

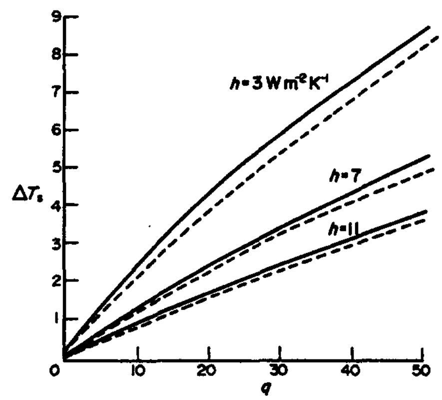

FIG. 9. Plot of ΔT_s vs q for $Ra = 40$ (solid lines) and 4000 (dotted lines) for $h = 3$, 7 and 11 W m^{-2} K^{-1}, respectively.

the only parameter still to be determind is the film coefficient, h.

The results from the one-dimensional model can be used as follows: ΔT_s is measured at a number of points on the surface, possibly by infra-red thermography. For each measured ΔT_s, the corresponding q is calculated using one of the curves shown in Fig. 9. The overall power dissipation causing hot spot(s) is then calculated by integrating q over the surface, i.e.

$$Q = \int_S q(\Delta T_s(\mathrm{x}))\,\mathrm{d}S \tag{12}$$

where S is the region of interest on the surface, and x is the position on S. Theoretically, the domain of integration should cover all areas where ΔT_s is nonzero. However, in practice a lower threshold value for ΔT_s may be set, depending on the application.

TEST OF THE MODEL

For each set of measured experimental results the power dissipation was estimated as shown above. For the cylindrical heater equation (12) takes the following form:

$$Q = 2\pi \int_0^{0.5} rq(\Delta T_s(r))\,\mathrm{d}r. \tag{13}$$

In doing these calculations three separate values of h (3.0, 7.0 and 11.0 W m^{-2} K^{-1}) were used and the results are presented in Table 2.

Simulated results were also obtained by using the model discussed earlier for a cylindrically symmetrical bed. The simulated bed had a radius of 10 m and a height of 10 m, and was composed of 0.01 m particles. The power source had both radius and height equal to 1.1 m, and was located in various positions. The power input was also varied. A value of h of 11 W m^{-2} K^{-1} was used for all of the simulations.

From the results in Table 3 it can be seen that the position and size of the source are also not important in estimating the power input from the surface temperatures and so we may conclude that this simplified model over the range of parameter values used, which is reasonable for coal stockpiles, is a useful one for monitoring self-heating.

The only major discrepancy could be caused by having an incorrect value of the surface heat transfer coefficient. Since in most cases measured surface temperatures are compared to those taken at an earlier time, the actual value of h is not too critical. Furthermore, values of h for different coal stockpiles are not likely to vary significantly, so that comparison between different piles should also be valid. Finally, once a body of measurements has been built up, including occasions where burning has actually occurred, one should be able to more readily interpret such results.

Table 2. Predicted energy input using the simplified model compared with experimental results

Q (W) input	Q (W) estimated	h (W m^{-2} K^{-1})	d_p (m)	H_q (m)
5.0	5.5	3.0	0.005	0.3
	11.0	7.0		
	16.5	11.0		
7.5	11.2	3.0	0.005	0.3
	21.6	7.0		
	31.6	11.0		
10.0	9.9	3.0	0.005	0.3
	18.9	7.0		
	27.5	11.0		
5.0	2.8	3.0	0.01	0.3
	5.6	7.0		
	8.6	11.0		
10.0	5.9	3.0	0.01	0.3
	11.0	7.0		
	15.6	11.0		
18.0	13.3	3.0	0.01	0.3
	23.8	7.0		
	32.4	11.0		
5.0	3.2	3.0	0.02	0.3
	6.2	7.0		
	9.3	11.0		
7.5	4.4	3.0	0.02	0.3
	8.4	7.0		
	12.4	11.0		
10.0	4.7	3.0	0.02	0.3
	8.7	7.0		
	12.2	11.0		

Table 3. Predicted energy input using the simplified model compared with model simulated results

	Q (W)	h (W m^{-2} K^{-1})	d_p (m)	H_q (m)
5.0	6.3	11.0	0.01	6.1
50.0	53.5	11.0		
500.0	507.6	11.0		
5.0	6.9	11.0	0.01	8.3
500.0	534.5	11.0		
5.0	4.8	11.0	0.01	2.8
50.0	50.4	11.0		

DISCUSSION

Real coal stockpiles and waste dumps tend to be very large and inhomogeneous and the task of modelling the dumps is well nigh impossible. If one could have a simple overall estimate of energy dissipation this could be very valuable for two reasons.

One would like to have such a measure of energy dissipation for burning dumps in order to try to estimate the amount of pollution, such as sulphur, being discharged into the atmosphere. If one knew the energy dissipation rate one could reasonably estimate the burning rate using a known heat of reaction and hence, through the known concentration of the material in the coal, the pollutant discharge rate.

Furthermore, one would also like to be able to use the results in this paper to estimate when spontaneous combustion in a dump or stockpile is becoming a problem so that the necessary preventive action, such as 'digging the hot spot out', could be taken. It is expensive and difficult to do this 'digging out', and one would like to be sure there was a real problem before taking such action, but on the other hand if one waits too long and the material is too hot, exposing it to the atmosphere is dangerous and the ensuing fire difficult to extinguish.

For monitoring of large dumps the only really feasible approach is to use infra-red thermography, preferably done from a helicopter. Only in this way is it really possible to identify 'temperature anomalies' in the surface temperatures at some points relative to the rest of the surface.

Experience has shown that it is essential to do such surveys under some sort of 'standard conditions' in order to interpret the results. It is most convenient to do these surveys on a still, cloudless night just before dawn. This is often done at full moon in order to ease the navigation problems involved.

In order to turn these surface temperature measurements into some sort of quantitative measure of the energy dissipation rate, it is clear one needs an estimate of the surface heat transfer coefficient. It is also clear from the analysis in the previous section that the result is fairly sensitive to the value of this parameter. Thus to effectively use the results in this paper it will be necessary to obtain some value for this quantity and preferably know how or if the value changes with the weather conditions.

The value of the heat transfer coefficient used by Brooks and Glasser [7] in their calculations was estimated by assuming that the main mechanism for heat loss from the surface (other than that removed by the flowing gas) was radiation. In this case assuming black body radiation to ambient temperature the value of the heat transfer coefficient one can calculate is about 6 $W m^{-2} K^{-1}$, which is entirely consistent with the estimated values in Table 1, but does not show the variations for the different particle sizes.

Brooks [18], in some experiments on an insulated coal bed open at both ends, found that his temperature–time results on both cooling and reaction runs could be fitted using heat transfer coefficients at the bottom and top of the bed with values of 3.7 and 9.3 $W m^{-2} K^{-1}$, respectively.

As seen from the values in Table 1, there is not a consistent variation of heat transfer coefficient with particle size, though the values for each particle size do not appear to vary with power input. Of course, different coals have very different surfaces ranging from bright shiny to dull matt and it could be expected, certainly, if radiation is important, that this alone would give rise to a range of values.

The problem is made worse by diurnal temperature variations being superimposed on the surface of the dump. However, as it is the difference between the 'anomaly' and the rest of the surface that is being measured, this effect is not as large as one might at first think. Obviously rain can also have a major effect on surface temperatures and wind has the dual effect of changing the surface heat transfer coefficient as well as forcing air through the dump, and so possibly changing the oxidation rate.

All one can say is that the larger the energy dissipation rate, the smaller the problems caused by these other effects; thus in these surveys one is looking for consistent patterns. It has often been observed when one is doing the measurement just before dawn that at such an 'anomaly' one first sees a 'surface cold spot' which is later followed at the same or a neighbouring place by the 'surface hot spot'. It would appear that this is caused by the cool air moving into the dump having a larger effect on surface temperatures in the early stage of self-heating than the hot air leaving it. This situation then rapidly changes over to a hot spot which then exhibits ever increasing surface temperatures.

In practice, dumps of different particle sizes show very different types of 'surface' anomalies. One can use this model to combine all the information into a single useful criterion by which one can decide when further confirmatory action is needed. This action usually takes the form of sending a person to the suspect area and knocking a temperature and oxygen probe a metre or two into the material, to confirm if there is a real problem.

In the case of burning dumps, again the manner of burning is very different for material of different particle sizes. For larger particles burning tends to take place much deeper in the dump and to be spread over a larger area than for smaller particles, where burning is often in the toe of the dump. Again the criterion can be used in combining all the results into a single measure of the burning or energy dissipation rate.

From this discussion it is clear that further work on estimating the surface heat transfer coefficient remains before these results can be used quantitatively, but on the other hand because of the insensitivity to other system parameters it does provide a basis for comparison between a wide variety of situations where the surface temperature patterns can be of quite a different nature. As a result of this it should prove possible in the future for the whole process of monitoring and protecting dumps and stockpiles to be done on a much more scientific basis.

In reality of course the heat source is not a heater but a section of the coal oxidizing (undergoing a chemical reaction). This fact should in no way significantly alter the conclusion as the flow and heat transfer mechanisms will be relatively unaffected by the chemical reaction. Furthermore, if all the oxygen is adsorbed the molar flow rate of the air will only be reduced by 20%. Conversely, if all the oxygen is turned into carbon monoxide the molar flow rate will be increased by 20% while if it is all turned to carbon dioxide the flow rate will remain unchanged with only a change in the mean molecular weight of the gas. All of these effects are essentially negligible relative to the purposes to which the model is intended to be used. Thus one can see that the model is directly applicable to self-heating situations in real stockpiles of coal.

CONCLUSIONS

In this paper by comparing experiments with a model it has been shown that one can adequately model thermally driven natural convection in a porous medium with an embedded heater. This can be done using the usual Boussinesq assumption and Darcy law as well as a heat transfer coefficient boundary condition at the free surface of the bed with a constant value of the heat transfer coefficient. This model fitted the experimental results for coal beds with nominal particle sizes from 0.005 to 0.02 m. The introduction of the constant heat transfer coefficient in the boundary condition in particular allows the surface temperatures to be adequately modelled.

In order to try to estimate power inputs in situations where beds have unknown particle size distributions with permeability variations with position as well as a lack of information on power sources, their positions and sizes, a simplified one-dimensional model was derived. Using this model as a basis it was shown that the power input could be predicted by the use of surface temperature measurements alone.

The only parameter to which this result showed sensitivity was the value of the heat transfer coefficient. When this result is to be used comparatively between different coal stockpiles or the same coal stockpile at different times, this did not pose a serious problem. However, in order to use this result quantitatively further work on the estimation of this parameter is required.

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APPENDIX

Application of an energy balance, the Boussinesq assumption (i.e. ρ_g is constant except where it appears in buoyancy terms, implying that v_z is assumed constant) and the Darcy law to the one-dimensional geometry shown in Fig. 8 yields the following equations, in dimensionless form:

$$\frac{\partial^2\Theta}{\partial\zeta^2} = v_z^* \frac{\partial\Theta}{\partial\zeta}. \tag{A1}$$

At the bottom of the bed, air flow is driven by the thermal convection induced by the power input q. It is assumed that no energy is lost from this end, and so the boundary condition can be written as

$$\left.\frac{\partial\Theta}{\partial\zeta}\right|_{\zeta=0} + v_z^*\Theta|_{\zeta=0} = \frac{qL}{k_b T_0}. \tag{A2}$$

Heat transfer at the free surface can be modelled by a constant heat transfer coefficient and so this boundary condition is given by

$$\left.\frac{-\partial\Theta}{\partial\zeta}\right|_{\zeta=1} = Bi\,\Theta|_{\zeta=1}. \tag{A3}$$

The general solution to equation (A1) is

$$\Theta(\zeta) = C_1 + C_2 \exp(\zeta v_z^*). \tag{A4}$$

Insertion of the boundary conditions gives the equation of the temperature profile, equation (11). However, the velocity term in (11) is still unknown. This may be calculated by applying the Darcy law and Boussinesq assumption

$$v_z = \frac{-K}{\mu}\left(\frac{\partial P}{\partial z}+\rho_g g\right) \quad \text{(Darcy's law)}. \tag{A5}$$

Substituting for the pressure gradient using the Boussinesq approximation gives

$$v_z = \frac{-K}{\mu}(-\rho_g^0 g+\rho_g g) = \frac{Kg\rho_g^0}{\mu}\left(1-\frac{T_0}{T}\right). \tag{A6}$$

Non-dimensionalizing equation (A6), and integrating both sides between $\zeta = 0$ and 1 (remembering that v_z is constant under the Boussinesq assumption) gives equation (10), which is the desired result.

CONVECTION THERMIQUE ET TEMPERATURE DE SURFACE DANS UN MILIEU POREUX

Résumé—Des mesures de températures stationnaires dans un milieu poreux qui contient un chauffoir noyé sont reproduites par un modèle thermique conventionnel. Un coefficient de transfert constant à la surface libre décrit convenablement la condition limite. On utilise un modèle monodimensionnel simple reliant la puissance d'alimentation à la température de surface sans tenir compte de la perméabilité, de la taille de la source ni de la profondeur. Il est montré que, pour les besoins pratiques, cela est cohérent vis-à-vis des expériences et du modèle général.

THERMISCHE KONVEKTION UND OBERFLÄCHENTEMPERATUREN AN PORÖSEN MEDIEN

Zusammenfassung—Die stationäre Temperaturverteilung in einem porösen Medium, in das ein Heizkörper eingebettet ist, wurde gemessen und mit einem üblichen Modell für die thermische Konvektion verknüpft. Es zeigt sich, daß ein konstanter Wärmeübergangskoeffizient an der freien Oberfläche die Randbedingung hinreichend gut beschreibt. Ein einfaches eindimensionales Modell wird entwickelt, das lediglich die freigesetzte Leistung mit der Oberflächentemperatur in Beziehung setzt und die Permeabilität, die Größe der Quelle und deren Tiefe außer Betracht läßt. Die Übereinstimmung zwischen Meß- und Rechenergebnissen ist brauchbar im Hinblick auf die beabsichtigte Anwendung des Modells zur Beobachtung der Selbsterwärmung in Lagerstätten.

ТЕПЛОВАЯ КОНВЕКЦИЯ И ТЕМПЕРАТУРЫ ПОВЕРХНОСТИ В ПОРИСТЫХ СРЕДАХ

Аннотация—Экспериментальные измерения установившейся температуры в пористой среде с помещенным в нее нагревателем приводятся в соответствие с общепринятой моделью тепловой конвекции. Показано, что постоянный коэффициент теплоотдачи на свободной поверхности дает адекватное описание граничных условий. Разработана простая одномерная модель, однозначно связывающая подводимую мощность с температурой поверхности независимо от проницаемости, размера теплового источника и глубины его погружения. Проиллюстрировано ее согласие с экспериментами и с более общей моделью, описывающей с требуемой точностью саморазогрев в засыпках хранилищ.